The Interaction and Merger of Nonlinear Internal Waves (NLIW)

PI: Darryl D. Holm
Mathematics Department
Imperial College London
180 Queen's Gate
SW7 2AZ London, UK
phone: +44 20 7594 8531 fax: +44 20 7594 8517 email: d.holm@ic.ac.uk

LONG-TERM GOALS

The long term goal of this project is the creation of a numerical model for nonhydrostatic, nonlinear oceanic internal waves for realistic geography and with general topography. The model is to be capable of integration with simpler process models and more complex oceanic models as a part of a multi-model coupled system.

OBJECTIVES

The objectives of this effort are:

1) Developing a multi-layer numerical code capability for modeling oceanic solitary waves and wave trains at the resolution of surface current fields.

2) Integrating and comparing results from the 2D internal wave code with large-scale, hydrostatic regional ocean models such as ROMS and fully three-dimensional small scale models such as SUNTANS.

3) Supporting the NLIWI remote sensing and in situ observational program through the provision of simulation data and surface wave signatures.

APPROACH

This project is a modelling/theoretical/numerical study of strongly interacting nonlinear internal waves. Assuming multi-layer columnar motion (MLCM) in two dimensions and following an Euler-Poincaré approach we derive a set of coupled equations describing the evolution of layer thicknesses and horizontal velocities in terms of these quantities alone. The model which results is fully nonlinear and contains a high order expansion of the nonhydrostatic terms present in the full Navier-Stokes equations. The free-surface signature is modelled directly and terms are included for prescribed topography. The equations are expressed naturally in terms of the evolution of layer momenta, with a coupled elliptic problem generating the complementary velocities. The layer model approach is particularly suitable for simulating the propagation of internal waves in areas such as the South China Sea since these waves generate little or no mixing, while propagating for extremely long distances.

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Key individuals:
During the past year research associate James Percival was supported by this grant.

WORK COMPLETED

The main technical results we achieved in the previous 2.5 years are, as follows:

☐ We derived the equations for multilayer columnar motion (MLCM) in two dimensions of a fluid with a free surface.

☐ We completed the design, construction and implementation of an unstructured-grid 2D numerical code for simulating NLIWI using the MLCM equations with several layers.

We are developing the 2D numerical code to solve the MLCM equations for a fluid consisting of an arbitrary number of layers. The MLCM numerical code will be used as a tool to explore the transmission and interaction of solitary waves in general domains with general topography in the presence or absence of rotation. In particular, the MLCM computes the free surface signature needed for coupling to surface wave models for retrievals from remote sensing data.

The MLCM code makes use of the finite element library provided by the Imperial College Ocean Model (ICOM, who are our close collaborators at Imperial) to allow maximum flexibility in formulating the discrete data structure, which is applied on unstructured triangular meshes. Our current implementation uses discontinuous linear finite elements for momentum, and continuous quadratic finite elements for velocity and layer thicknesses. The code makes use of the PETSc iterative solvers library (http://www-unix.mcs.anl.gov/petsc/petsc-as/) which will allows flexibility of choices of solvers and preconditioners (including algebraic multigrid) as well as future development of parallel implementations.

For initialization and verification of the 2D FEM Model we are developing in parallel a 1D model in finite volume discretization as well as a symplectic integrator for the shooting problem involved in finding the family of numerical solitary and periodic traveling wave solutions to the MLCM equations for an arbitrary number of layers and for a general background stratification. The 1D MLCM model will also be useful for comparing MLCM results with results of other NLIWI PIs.

RESULTS

Development of the 2D finite element code has focused on issues related to numerical stability. Thanks to the extreme flexibility of the finite element code available to us we have examined a number of discretizations for the model, currently using discontinuous linear elements for momentum, to allow the representation of steep fronts. Meanwhile, the layer thicknesses and velocities are represented using a continuous quadratic Galerkin formulation for maximum accuracy and consistency within the equations, while remaining LLB stable. (LBB stable is a standard finite-elements condition meaning that the approximate solution of the finite element problem converges to the true solution as the computational mesh is made finer.)

Figure 1 shows an example of the use of these disparate element choices in modeling the propagation of a solitary wave in the two-layer case.

Experiments with the one-dimensional code show the suitability of the layer approach to area of internal wave interaction. Figure 2 shows the evolution of two identical solitary waves, which are involved in a head on collision and exchange their momentum. The solitary waves interact elastically, retaining their shape and amplitude following the interaction but have obtained velocities of the opposite sign. Diffusion in the experiment was limited to the numerical effects provided by the QUICK upwinding scheme used for the tracer variables. Note that the left-hand plot shows precisely the derived variation in the free surface, which is required for surface wave models to assimilate SAR altimetry data.
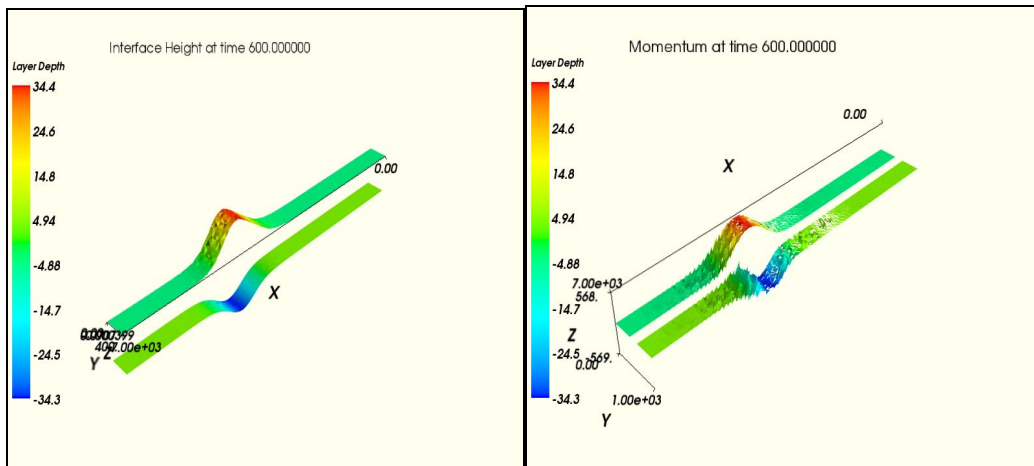

Figure 1 Examples of continuous height fields and discontinuous momenta from a two layer run of a travelling wave solution the MLCM code.
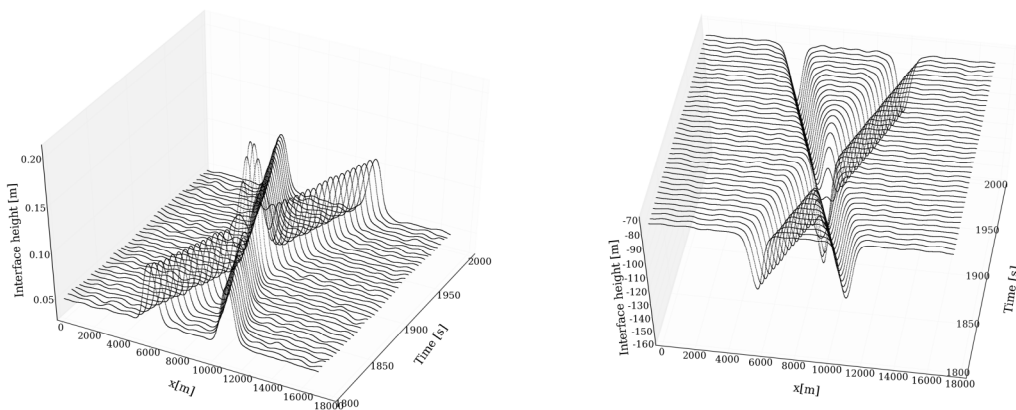

Figure 2 Plot showing the evolution of a) the free surface and b) the density interface under the interaction of two travelling waves of opposite polarity in the 1D MLCM code.

The multilayer system contains a barotropic mode which, unless specifically addressed, tends to reduce the range of stable timesteps for conditionally stable time stepping methods so severely as to limit the usefulness of the code. The linear dispersion relation for a two-layer system is shown in Figure 3. This has two modes, corresponding for short waves to the dispersion relations for a linear external barotropic mode and an internal baroclinic mode respectively. For a basin of depth 1000m and assuming a horizontal resolution of 100m this implies timesteps of the order of one second would be necessary to adequately resolve the baroclinic mode. Work is ongoing for a barotropic/baroclinic splitting method to separate out the fast mode and integrate it either implicitly, or on a shorter timescale.
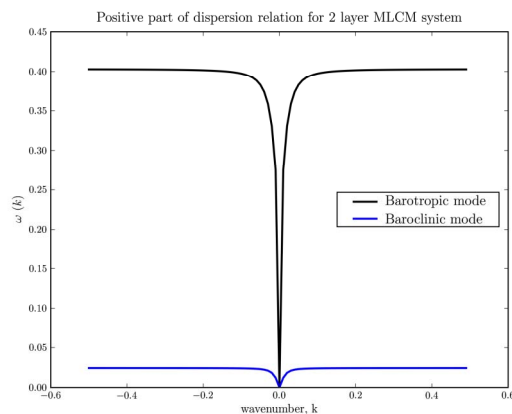

Figure 3 Graph of the linear dispersion relation from the MLCM code for the case d_1=100m, d_2=200m, ρ_1/ρ_2=(1023/1026), showing the separation of the barotropic and baroclinic modes.

We plan to:

☐ Continue improving and developing our unstructured grid 2D numerical code into an integrated tool for real-time simulation of nonlinear wave interactions in realistic geographies.

☐ Undertake benchmarking and validation studies for numerical modelling of wave interactions in two and three layer systems.

☐ Interface our free surface signature data with other remote sensing efforts as part of the NLIWI program.

IMPACT/APPLICATIONS

The primary potential future impact of these results is on the design and use of a new tool for predicting the development of a field of interacting internal waves in a domain of realistic geography. This will provide the US Navy with an efficient tool for prediction of the internal

wave field in a particular domain based on a minimal set of field measurements and SAR altimetry data.

RELATED PROJECTS

This project forms part of the ONR NLIWI DRI. Our primary related partners in this effort are Robert Street, Oliver Fringer and the rest of the SUNTANS team at Stanford. In addition David Farmer and Karl Helfrich are developing an internal wave code. The MLCM code provides for verification and validation with the results of regional models, such as those of Gallacher, Ko and McWilliams. Our partners in the surface signature analysis for remote sensing are Wooyoung Choi, David Lyzenga, Norm Malinas and Chris Wackerman. Finally there is scope for integration with Chris Jackson's empirical phase propagation model.